RESTORATION OF OPTICAL PATTERNS

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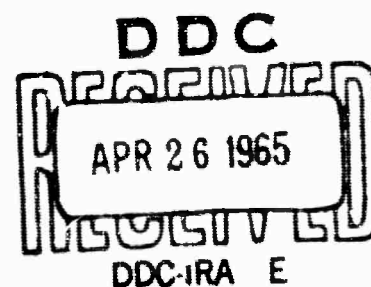
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NOTICES

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Abstract

This paper treats the problem of estimating the intensity distribution of an optical pattern which has been distorted by diffraction and noise. Under the assumption that the signal and noise are additive and that they have prescribed means and covariance matrices, the optimum linear estimate of the object distribution is obtained. This estimate is shown to be a linearly-filtered combination of the a priori mean and the observed image distribution, with the details of the filtering depending on the prior information, the noise statistics, and the optical imaging system. The performance of the optimum estimation procedure is evaluated for the important special case of large a priori uncertainty. Finally, the optimum procedure for processing multiple observations is discussed. It is shown that as the number of observations becomes large, the estimate approaches the true object.

I. Introduction

The problem of restoring the detail removed from an optical pattern
by diffraction has been considered by Harris [1]. Using some well-
known results from the theory of Fourier transforms, he showed that
in the absence of noise, an optical pattern known a priori to be of bounded
spatial extent can be exactly reconstructed despite the rejection of high
spatial frequencies by an optical system.

While the noise-free problem is of considerable theoretical interest,
it does not accurately represent the physical world. In any actual
situation, spurious disturbances such as background radiation, film
granularity, and detector noise are superimposed on the quantities to be
measured. Thus, any restoration procedure will involve a certain
amount of unavoidable error. Furthermore, a given restoration procedure
may perform very badly in the presence of noise even though it repro-
duces the pattern exactly when noise is absent. Hence, it is important
to take into account any knowledge we have about the noise statistics, as
well as the a priori information about the pattern to be estimated, in
designing a restoration procedure.

In this paper, we treat the problem of obtaining optimum estimates
of an optical object in the presence of additive noise using the criterion
of minimum mean square error.

II. Statement of the Problem

Throughout the paper, we assume for convenience that we are dealing with incoherent illumination. All observed and estimated quantities are intensities, and the transfer function of the optical system is that appropriate for incoherent light [2]. This assumption simplifies the discussion somewhat and avoids confusion, but it is by no means essential.

The object to be estimated is denoted by $x(\alpha, \beta)$, where α and β are the object-plane coordinates. For simplicity of presentation, it is assumed that the optical system exhibits spatial invariance [3], and that it has a point spread function $a(\xi, \eta)$ where ξ and η are the image-plane coordinates. The configuration is illustrated in Figure 1.

The image of $x(\alpha, \beta)$ is given by

$$y(\xi, \eta) = x(\xi, \eta) * a(\xi, \eta) \tag{1}$$

where $x(\xi, \eta)$ is the object distribution referred to the image plane and the star denotes convolution. This image is corrupted by additive noise $n(\xi, \eta)$, and the resultant image $z(\xi, \eta) = y(\xi, \eta) + n(\xi, \eta)$ is available for processing. The objective is to estimate $x(\alpha, \beta)$ based on the ob-

servation $z(\xi, \eta)$ and any a priori information available about the object.

III. Sampling and Vector Representation

Almost any reasonable object $x(\alpha, \beta)$ can be represented to the desired degree of accuracy by partitioning the object plane into small regions over which $x(\alpha, \beta)$ is essentially constant. If these regions are made sufficiently small, each of them can be treated as an impulse or point source of light. The image due to one of these regions is then approximately proportional to the point spread function of the optical system displaced according to the location of that region. The total image is the superposition of the images of all the elementary regions.

Let the intensity of the k-th region in the object plane be $x_k = x(\alpha_k, \beta_k)$. The image of this region is approximately $x_k a(\xi - \xi_k, \eta - \eta_k)$, where ξ_k and η_k are the image-plane coordinates corresponding to α_k and β_k. If the object consists of n such regions, the total image is

$$y(\xi, \eta) = \sum_{k=1}^{n} x_k a(\xi - \xi_1, \eta - \eta_k) \tag{2}$$

If this image is sampled at the points $(\xi^1, \eta^1), \ldots, (\xi^m, \eta^m)$, we have

$$y(\xi^j, \eta^j) = \sum_{k=1}^{n} x_k \, a(\xi^j - \xi_k, \eta^j - \eta_k) \qquad (3)$$

Note that (ξ^j, η^j) is not necessarily equal to (ξ_j, η_j). If we set $a_{jk} = a(\xi^j - \xi_k, \eta^j - \eta_k)$ and $y_j = y(\xi^j, \eta^j)$, we have

$$y_j = \sum_{k=1}^{n} x_k \, a_{jk} \qquad (4)$$

In matrix form,

$$\underline{y} = \underline{A}x \qquad (5)$$

where

$$\underline{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix} \qquad \underline{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$\underline{A} = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$$

The samples y_j are in general perturbed by noise samples n_j, and

the observed vector is

$$\underline{z} = \underline{y} + \underline{n} = \underline{A}\underline{x} + \underline{n} \qquad (\text{v})$$

where $n = \begin{pmatrix} n_1 \\ \cdot \\ \cdot \\ n_m \end{pmatrix}$

Throughout the remainder of the paper, it is assumed that the approximate vector representation discussed above is sufficiently accurate that the associated error can be neglected. Hence, the vector $\underline{x}$ will be regarded as the object, and the estimation procedures considered will be evaluated in terms of how closely they reproduce $\underline{x}$.

IV. The Optimum Estimation Procedure

In order to proceed further, we now make some additional assumptions about the noise and the a priori information. We assume that the noise vector $\underline{n}$ has zero mean and known covariance matrix $\underline{K}_n$. The restriction to zero mean is unimportant, since only the fluctuations about the mean limit system performance. Further, we assume that the a priori information about the object to be estimated consists of the mean vector $\underline{\mu}$ and the covariance matrix $\underline{K}_x$, and that $\underline{x}$ and $\underline{n}$ are independent. The quantities $\underline{\mu}$ and $\underline{K}_x$ do not necessarily correspond to any actual statistical

fluctuations which the object undergoes, but rather reflect our initial ignorance of the object. Roughly speaking, a covariance matrix K_x with small entries corresponds to a situation in which we have considerable prior information about the object, while a covariance matrix with large entries corresponds to a situation in which we have little prior information.

The approach to the estimation problem taken in this paper is that of discrete Wiener filtering of the perturbed image samples. That is, we seek that discrete linear filter whose output $\hat{x}$ minimizes the mean square error $E[(\hat{x} - x)' (\hat{x} - x)]$, where the prime denotes vector or matrix transpose.

We first assume that the prior mean μ is equal to zero. The linear filtering operation on the observed vector z is then represented by matrix multiplication, and any linear estimate of x is a vector of the form $H z$. Our objective is to choose the matrix H to minimize the mean square error

$$\partial = E[(H z - x)' (H z - x)] \tag{7}$$

Using the fact that $z = A x + n$, this expression can be written in the form

$$e = E\left\{ [(\underline{H}\,\underline{A} - \underline{I})\,\underline{x}]'\,[(\underline{H}\,\underline{A} - \underline{I})\,\underline{x}] \right.$$
$$+ [(\underline{H}\,\underline{A} - \underline{I})\,\underline{x}]'\,\underline{H}\,\underline{n} + (\underline{H}\,\underline{n})'\,(\underline{H}\,\underline{A} - \underline{I})\,\underline{x}$$
$$\left. + \;(\underline{H}\,\underline{n})'\,\underline{H}\,\underline{n} \right\}$$

Since $\underline{x}$ and $\underline{n}$ are assumed to be independent random vectors with zero means, the cross-product terms in (8) are zero and the error expression reduces to

$$e = E\left\{ [(\underline{H}\,\underline{A} - \underline{I})\,\underline{x}]'\,[(\underline{H}\,\underline{A} - \underline{I})\,\underline{x}] + (\underline{H}\,\underline{n})'\,\underline{H}\,\underline{n} \right\}$$

$$= tr\left[(\underline{H}\,\underline{A} - \underline{I})\,\underline{K}_x\,(\underline{H}\,\underline{A} - \underline{I})' + \underline{H}\,\underline{K}_n\,\underline{H}' \right] \tag{9}$$

where tr denotes the trace of a matrix [4].

Upon expanding (9) and completing the square, we obtain an expression for e which can be minimized by inspection.

$$e = tr\left\{ [\underline{H}(\underline{A}\,\underline{K}_x\,\underline{A}' + \underline{K}_n)^{\frac{1}{2}} - \underline{K}_x\,\underline{A}'\,(\underline{A}\,\underline{K}_x\,\underline{A}' + \underline{K}_n)^{-\frac{1}{2}}] \right.$$
$$[\underline{H}(\underline{A}\,\underline{K}_x\,\underline{A}' + \underline{K}_n)^{\frac{1}{2}} - \underline{K}_x\,\underline{A}'\,(\underline{A}\,\underline{K}_x\,\underline{A}' + \underline{K}_n)^{-\frac{1}{2}}]'$$
$$\left. + \underline{K}_x - \underline{K}_x\,\underline{A}'\,(\underline{A}\,\underline{K}_x\,\underline{A}' + \underline{K}_n)^{-1}\,\underline{A}\,\underline{K}_x \right\} \tag{10}$$

where $(\underline{B})^{\frac{1}{2}}$ is the symmetric square root of the symmetric matrix $\underline{B}$ [5]. Since the term involving $\underline{H}$ in (10) is non-negative, the mean square error e achieves its minimum for that $\underline{H}$ which satisfies

$$[\underline{H}(\underline{A}\,\underline{K}_x\underline{A}' + \underline{K}_n)^{\frac{1}{2}} - \underline{K}_x\underline{A}' \, (\underline{A}\,\underline{K}_x\underline{A}' + \underline{K}_n)^{-\frac{1}{2}}] = 0$$

This yields as the optimum discrete linear filter for the zero-mean case

$$\underline{H} = \underline{K}_x\underline{A}' \, (\underline{A}\,\underline{K}_x\underline{A}' + \underline{K}_n)^{-1} \tag{11}$$

A somewhat more convenient form for our purposes, obtained after some manipulation of matrices, is

$$\underline{H} = (\underline{A}'\,\underline{K}_n^{-1}\,\underline{A} + \underline{K}_x^{-1})^{-1}\,\underline{A}'\,\underline{K}_n^{-1} \tag{12}$$

Thus far we have assumed that the prior mean $\underline{\mu}$ is zero. The above results are now extended to the general case in which $\mu \neq 0$. This extension is accomplished by noting that the above results are applicable to estimating the amount by which $\underline{x}$ deviates from $\underline{\mu}$, namely $\underline{x} - \underline{\mu}$. Hence, we merely apply the above result to $\underline{x} - \underline{\mu}$ and add $\underline{\mu}$ to the resulting estimate of the deviation. This yields as the optimum linear estimate of th object distribution

$$\overset{\wedge}{\underline{x}} = \underline{\mu} + \underline{H}(\underline{z} - \underline{A}\underline{\mu})$$
$$= \underline{H}\underline{z} + (\underline{I} - \underline{H}\underline{A})\underline{\mu} \tag{13}$$

which becomes upon substituting (12) for $\underline{H}$

$$\hat{\underline{x}} = (\underline{A}'\,\underline{K}_n^{-1}\,\underline{A} + \underline{K}_x^{-1})^{-1}\,\underline{A}'\,\underline{K}_n^{-1}\,\underline{z}$$

$$+ (\underline{A}'\,\underline{K}_n^{-1}\,\underline{A} + \underline{K}_x^{-1})^{-1}\,\underline{K}_x^{-1}\,\underline{\mu} \qquad (14)$$

This is the desired general result.

V. Special Cases

It is seen that the optimum estimate $\hat{\underline{x}}$ is just a linearly-filtered combination of the observed vector $\underline{z}$ and the *a priori* mean $\underline{\mu}$. The details of this filtering and combining depend on the noise, the optical system, and the *a priori* information. Since the operations involved in obtaining the optimum estimate are in general rather complicated, we now consider several special cases in order to gain additional insight into the estimation procedure.

A. $\underline{K}_n = \sigma_n^2\,\underline{I}$, $\underline{K}_x = \sigma_x^2\,\underline{I}$

To begin with, we assume that $\underline{K}_n = \sigma_n^2\,\underline{I}$ and $\underline{K}_x = \sigma_x^2\,\underline{I}$, where $\underline{I}$ is the identity matrix. This corresponds to a situation in which the noise is white and in which information about one coordinate of $\underline{x}$ provides no information about any other coordinate. In this case, the

optimum estimate becomes

$$\underline{x} = (\frac{A'A}{\sigma_n^2} + \frac{I}{\sigma_x^2})^{-1} \frac{A'}{\sigma_n^2} \underline{z}$$

$$+ (\frac{A'A}{\sigma_n^2} + \frac{I}{\sigma_x^2})^{-1} \frac{1}{\sigma_x^2} \underline{\mu} \tag{15}$$

This expression provides some insight regarding the relative
weighting of the observation $\underline{z}$ and the a priori mean $\underline{\mu}$. If the a priori
uncertainty about $\underline{x}$ is much larger than the uncertainty due to the
additive noise (the usual case), then $\sigma_x^2 >> \sigma_n^2$. From (15) we see
that in this case the optimum estimation procedure weights the ob-
servation $\underline{z}$ much more heavily than the a priori mean $\underline{\mu}$ provided that
the elements of $\underline{A}$ are not too small. This agrees with intuition, since
if the a priori uncertainty is very large we have little confidence in the
a priori mean. On the other hand, if $\sigma_n^2 >> \sigma_x^2$, we have a very noisy
measurement. From (15), we see that in this case $\underline{\mu}$ is weighted more
heavily than $\underline{z}$, again in agreement with intuition, and the experiment
is not a very informative one.

B.   Large a priori uncertainty

We can obtain additional insight into the optimum estimation procedure by looking at the important limiting case of large a priori uncertainty. The optimum estimate in this case is obtained by making a Neumann series expansion [6] of (14) and retaining only the first term. This procedure yields

$$\hat{\underline{x}} = (\underline{A}' \underline{K}_n^{-1} \underline{A})^{-1} \underline{A}' \underline{K}_n^{-1} \underline{z} \qquad (16)$$

This procedure is now considered under various assumptions about $\underline{A}$ and $\underline{K}_n$.

If the system matrix $\underline{A}$ is square and invertible, (16) becomes

$$\hat{\underline{x}} = (\underline{A}' \underline{K}_n^{-1} \underline{A})^{-1} \underline{A}' \underline{K}_n^{-1} \underline{A}\underline{A}^{-1} \underline{z} = \underline{A}^{-1} \underline{z} \qquad (17)$$

In this case, the optimum estimation procedure merely "divides out" the effects of the optical system by multiplying the observed vector by $\underline{A}^{-1}$. This is precisely the procedure which would be followed in the absence of noise, and the above analysis clearly indicates the limitations on this procedure and the conditions under which it is optimum. These conditions are negligible a priori information about the object to be estimated (except for information regarding spatial extent) and a

sampling scheme in the image plane which yields a square and invertible system matrix $\underline{A}$. If either of these conditions is violated, the more general data processing procedures given by (14) or (16) should be used.

A further simplification of (16) is achieved in the event that the additive noise is white. In this case, $\underline{K}_n = \sigma_n^2 \underline{I}$ and

$$\hat{\underline{x}} = (\underline{A}' \underline{A})^{-1} \underline{A}' \underline{z} \tag{18}$$

This expression reduces to (17), of course, if A is square and invertible.

VI. Evaluation of the Optimum Procedure

For the purpose of evaluating the optimum estimation or restoration procedure, we assume that the actual (but unknown) object vector is $\underline{x}_o$. The observed vector is thus

$$\underline{z} = \underline{A} \underline{x}_o + \underline{n}$$

We further assume that the a priori uncertainty is very large, which leads us to use the estimate given by (16). This yields

$$\hat{\underline{x}} = (\underline{A}' \underline{K}_n^{-1} \underline{A})^{-1} \underline{A}' \underline{K}_n^{-1} (\underline{A} \underline{x}_o + \underline{n})$$

$$= \underline{x}_o + (\underline{A}' \underline{K}_n^{-1} \underline{A})^{-1} \underline{A}' \underline{K}_n^{-1} \underline{n} \qquad (19)$$

The mean square error (conditioned on the actual object vector $\underline{x}_o$) is given by

$$e = E[(\hat{\underline{x}} - \underline{x}_o)' (\hat{\underline{x}} - \underline{x}_o)]$$

$$= E\left\{ [(\underline{A}' \underline{K}_n^{-1} \underline{A})^{-1} \underline{A}' \underline{K}_n^{-1} \underline{n}]' [(\underline{A}' \underline{K}_n^{-1} \underline{A})^{-1} \underline{A}' \underline{K}_n^{-1} \underline{n}] \right\}$$

which after some manipulation of matrices can be written

$$e = \text{tr}\left[(\underline{A}' \underline{K}_n^{-1} \underline{A})^{-1} \right] \qquad (20)$$

In the special case of white noise, $\underline{K}_n = \sigma_n^2 \underline{I}$ and

$$e = \sigma_n^2 \, \text{tr}\left[(\underline{A}' \underline{A})^{-1} \right] \qquad (21)$$

Recall that the terms in the matrix $\underline{A}$ are determined by the point spread function of the optical system and the sampling points. Similarly, the covariance matrix $\underline{K}_n$ is determined by the sampling scheme in the image plane as well as by the nature of the noise. Hence, it is clear that we can control the form of the matrix $\underline{A}' \underline{K}_n^{-1} \underline{A}$ to a limited extent

through our choice of sampling schemes. A judicious choice of sampling points can reduce the mean square error associated with the optimum estimation procedure.

The problem of where to sample and how many samples to take is both important and nontrivial, and is currently being investigated. Although no general solution is available at this point, some of the features of this solution are apprent. First, it is clear that samples near the central part of the image are more useful than samples on the fringes where the signal-to-noise ratio is low. Second, it is not difficult to show that adding additional samples without changing the location of previous samples will never increase the error, and in fact will decrease it except in special cases. Thus, it may well be that we will want to take more samples than we have unknowns to estimate $(m > n)$. The usefulness of additional samples is limited by the increase in the complexity of the estimation procedure, of course, since the

mum estimate for th case is given by (16) where $\underline{A}$ is an m x n matrix rather than by the intuitive expression (17) with $\underline{A}$ an n x n matrix.

VII. Example

We now consider a simple example to illustrate several of the above ideas. The configuration for this example is shown in Figure 2. The object is a single line source at the origin in the object plane, and it is desired to estimate the intensity of this source. We assume in all cases that the a priori uncertainty is very large. The aperture is a rectangular slit with point spread function

$$a(\xi) = a \ \text{sinc}^2 \xi$$
$$= a \ \frac{\sin^2 \pi\xi}{(\pi\xi)^2} \tag{22}$$

The noisy image is then given by

$$y(\xi) = x a \ \text{sinc}^2\xi + n(\xi) \tag{23}$$

To begin with, we consider the case where a single sample is taken at $\xi = 0$. We then have

$$z = a x + n \tag{24}$$

The matrix $\underline{A}$ in this case reduces to the scalar a, and the optimum estimate is, from (17)

$$\hat{x} = z/a \tag{25}$$

The mean square error associated with this estimate is, from (21),

$$e = \frac{\sigma_n^2}{a^2} \tag{26}$$

where σ_n^2 is the variance of the noise sample at $\xi = 0$.

Now suppose that in addition to the sample at $\xi = 0$, a second sample is taken at $\xi = 1/2$. Assume for the moment that $\underline{K}_n = \sigma_n^2 \underline{I}$; i.e., the noise samples are uncorrelated and have the same variance. The system matrix $\underline{A}$ in this case becomes

$$\underline{A} = \begin{pmatrix} a \\ 4a/\pi^2 \end{pmatrix} = \begin{pmatrix} a \\ 0.404\, a \end{pmatrix}$$

Since $\underline{A}$ is no longer square, (16) rather than (17) must be used to estimate x. The optimum estimate for this case is

$$\hat{x} = 0.859\, z_1/a + 0.347\, z_2/a \tag{27}$$

where z_1 is the observed sample at the origin and z_2 that at $\xi = 1/2$.

The mean square error associated with this estimate is seen from (21) to be

$$e = 0.859 \, \sigma_n^2 / a^2, \tag{28}$$

a moderate reduction over that associated with the single-sample procedure.

To illustrate the effects of correlation between noise samples in a particular case, we consider the same two sampling points as above, but now assume

$$K_n = \sigma_n^2 \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}$$

The optimum estimate is seen from (16) to be

$$\hat{x} = 1.052 \, z_1 / a - 0.127 \, z_2 / a \tag{29}$$

with an associated mean square error

$$e = 0.989 \, \sigma_n^2 / a^2 \tag{30}$$

The effects of correlation between noise samples in this example may be assessed by comparing (29) and (30) with (27) and (28).

VIII. Multiple Observations

In many situations, we make not one but several observations of the object to be estimated. For example, we may have several different photographs of the object. If the noise samples associated with different observations are independent, we can make use of this fact to improve our estimate.

Suppose that q observations $\underline{z}^1 \,..\, \underline{z}^q$ are made, where

$$\underline{z}^i = \underline{A}\,\underline{x} + \underline{n}^i$$

The $\underline{n}^i$ are assumed to independent noise vectors, each having zero mean and covariance matrix $\underline{K}_n$. The optimum estimate based on the observations $\underline{z}^1 \,..\, \underline{z}^q$ can be shown by a method similar to that used for a single observation to be

$$\underline{x}_q = (q\underline{A}'\underline{K}_n^{-1}\underline{A} + \underline{K}_x^{-1})^{-1}\, q\underline{A}'\underline{K}_n^{-1}\underline{\bar{z}}$$

$$+ (q\underline{A}'\underline{K}_n^{-1}\underline{A} + \underline{K}_x^{-1})^{-1}\underline{K}_x^{-1}\underline{\mu} \tag{31}$$

where

$$\underline{\bar{z}} = \frac{1}{q}\,\sum_{i=1}^{q}\underline{z}^i \tag{32}$$

is the sample mean of the observed vectors. This estimate is identical in form to that given by (14) for a single observation, and it reduces to (14) for $q = 1$ as it should.

If the matrix $\underline{A}' \underline{K}_n^{-1} \underline{A}$ is non-singular, as it should be for good restoration, we can consider the limiting value of the estimate $\hat{\underline{x}}_q$ as the number of observations becomes arbitrarily large. We have

$$\lim_{q \to \infty} \hat{\underline{x}}_q = (\underline{A}' \underline{K}_n^{-1} \underline{A})^{-1} \underline{A}' \underline{K}_n^{-1} \bar{\underline{z}} \tag{33}$$

But by the law of large numbers [7], we further have

$$\lim_{q \to \infty} \bar{\underline{z}} = \underline{A} \underline{x}_0 \tag{34}$$

with probability one, where $\underline{x}_0$ is the true value of the object, assumed to be constant throughout the experiment. The optimum estimate thus reduces in the limit to

$$\lim_{q \to \infty} \hat{\underline{x}}_q = (\underline{A}' \underline{K}_n^{-1} \underline{A})^{-1} \underline{A}' \underline{K}_n^{-1} \underline{A} \underline{x}_0 = \underline{x}_0 \tag{35}$$

and we see that the object is reproduced exactly despite the presence of noise.
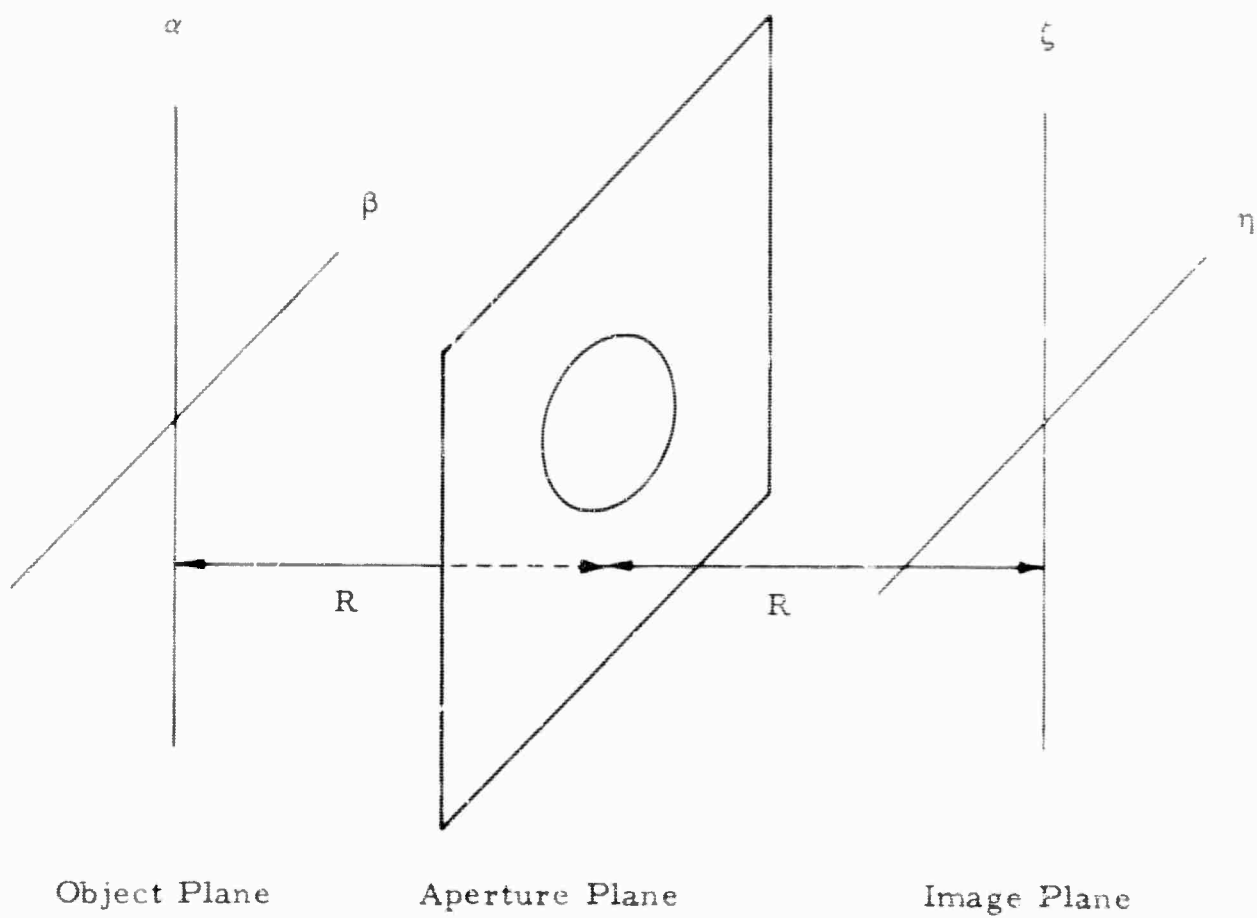
Figure 1. Optical imaging configuration.

Object   Aperture

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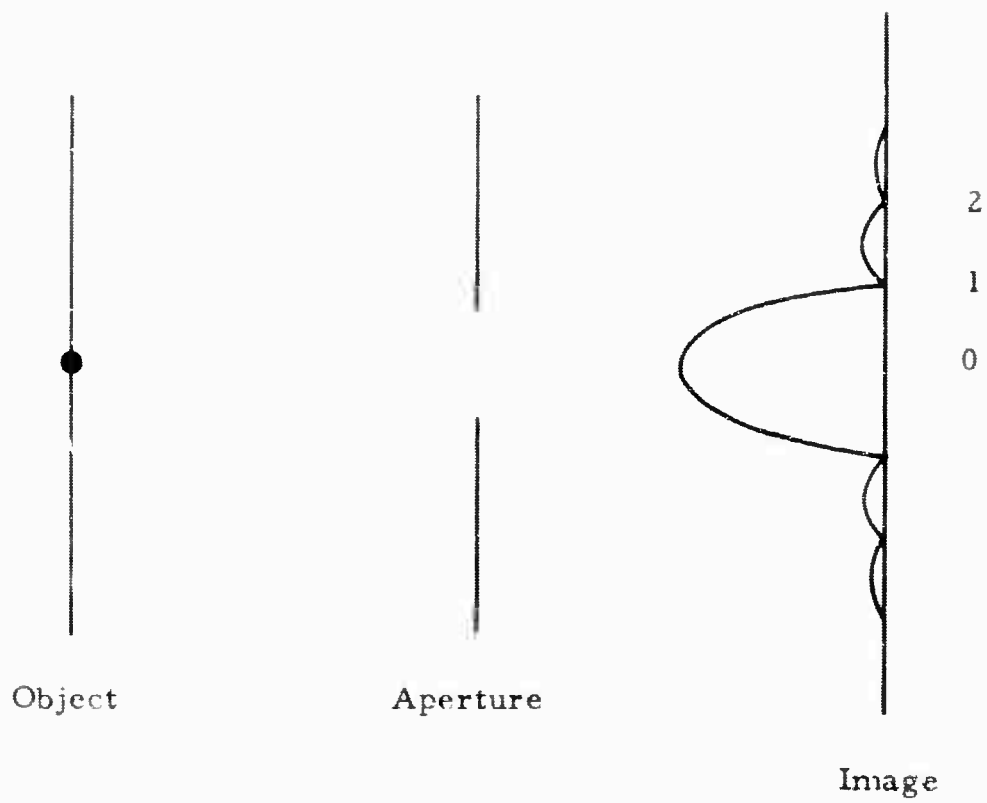
Image

Figure 2.   Image of a line source through a rectangular aperture.

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13. ABSTRACT   This paper treats the problem of estimating the intensity distribution of an optical pattern which has been distorted by diffraction and noise. Under the assumption that the signal and noise are additive and that they have prescribed means and covariance matrices, the optimum linear estimate of the object distribution is obtained. This estimate is shown to be a linearly-filtered combination of the a priori mean and the observed image distribution, with the details of the filtering depending on the prior information, the noise statistics, and the optical imaging system. The performance of the optimum estimation procedure is evaluated for the important special case of large a priori uncertainty. Finally, the optimum procedure for processing multiple observations is discussed. It is shown that as the number of observations becomes large, the estimate approaches the true object.

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